Dynamic decision making and value computations in medial frontal cortex

Bilal A. Bari and Jeremiah Y. Cohen*

The Solomon H. Snyder Department of Neuroscience, Brain Science Institute, Kavli Neuroscience Discovery Institute, Johns Hopkins University, Baltimore, MD, United States
*Corresponding author: e-mail address: Jeremiah.cohen@jhmi.edu

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Abstract

Dynamic decision making requires an intact medial frontal cortex. Recent work has combined theory and single-neuron measurements in frontal cortex to advance models of decision making. We review behavioral tasks that have been used to study dynamic

decision making and algorithmic models of these tasks using reinforcement learning theory. We discuss studies linking neurophysiology and quantitative decision variables. We conclude with hypotheses about the role of other cortical and subcortical structures in dynamic decision making, including ascending neuromodulatory systems.
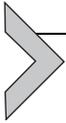
1. Introduction

Nervous systems evolved in highly dynamic environments. Adaptive behavior in the natural world requires not just learning which actions improve survival, but also changing behavior as the environment changes. This describes an active feedback process in which organisms interact with the environment through actions, learn from feedback, and adjust future actions adaptively. The ability to behave flexibly is a ubiquitous feature of life, ranging from flies (Ofstad, Zuker, & Reiser, 2011) to humans.[a] Despite the ubiquity of flexible decision making, it is a historically under-studied problem in systems neuroscience. Progress in reinforcement learning over the past two decades has provided a biologically plausible and mathematically sophisticated framework for studying these problems (Bertsekas & Tsitsiklis, 1996; Sutton & Barto, 1998). Parallel progress in tool development in mice has enabled the dissection of neural circuits needed for detailed biological insight (Luo, Callaway, & Svoboda, 2018). The intersection of these two fields—reinforcement learning and neural circuit dissection— holds promise to further our algorithmic- and implementation-level understanding of cognition.

This chapter reviews behavioral assays for investigating value-guided behavior, explores biologically plausible algorithms of value-based decision making, and ends with an overview of the neural systems thought to instantiate these functions. We highlight recent evidence demonstrating value computations in the medial prefrontal cortex (mPFC) and cortico-basal-ganglia loops. Given the myriad decision making dysfunctions seen in patients with mental illnesses, understanding its neural basis on multiple levels is crucial for developing targeted therapies (Cáceda, Nemeroff, & Harvey, 2014).

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[a] It has even been observed in organisms without nervous systems (pea plants; Dener, Kacelnik, & Shemesh, 2016).
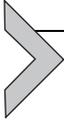
2. Value-based decision making

Decision making is at the heart of many fields, including economics, political science, psychology, engineering, medicine, and neuroscience. A decision is defined as a deliberative process that results in commitment to a categorical proposition (Gold & Shadlen, 2007). Decisions are the result of integrating both external (e.g., sensory) as well as internal evidence (e.g., predictions). These two domains have largely been studied in the context of perceptual (i.e., external evidence) and value-based (i.e., internally generated evidence) decision-making tasks (Gold & Shadlen, 2007; Yu, 2014). Within each domain, quantitative mathematical frameworks have provided semantically meaningful interpretations of neural activity in key brain structures. A key concept is the decision variable, which "*represents the accrual of all sources of priors, evidence, and value into a quantity that is interpreted by the decision rule to produce a choice*" (Gold & Shadlen, 2007). The decision variable represents the common currency used by the brain to generate a single categorical action among all possibilities.

Value-based decision making tasks require subjects to choose on the basis of expected utility or subjective value (Glimcher, Dorris, & Bayer, 2005; Sanfey, Loewenstein, McClure, & Cohen, 2006; Sugrue, Corrado, & Newsome, 2005). Typically, sensory stimuli are salient, which minimizes perceptual uncertainty. Decisions are made on the basis of values learned over long time periods (Morris, Nevet, Arkadir, Vaadia, & Bergman, 2006) or values learned over short timescales (Lau & Glimcher, 2005; Sugrue, Corrado, & Newsome, 2004; Tsutsui, Grabenhorst, Kobayashi, & Schultz, 2016). Importantly, these values are not sensory properties of stimuli (like brightness or contrast) but are internal variables that must be learned through experience. Compared to perceptual tasks, the subjective nature of value-based tasks makes them much more difficult to control. However, rigorous formalisms from the fields of economics (Rangel, Camerer, & Montague, 2008) and reinforcement learning (Sutton & Barto, 1998) have greatly benefited the study of value-based decisions. As such, these tasks are well-poised to address questions about representations of cognitive information.

2.1 Pavlovian systems

Pavlovian behavior describes innate, reflexive behavioral responses to stimuli that have been assigned value (Rescorla, 1988). The phenomenon was discovered by Ivan Pavlov while conducting studies of the digestive system

in dogs (Pavlov & Anrep, 1928).[b] In the case of reward, the Pavlovian response is preparatory (e.g., approach) and consummatory. In the case of punishment, the Pavlovian response is avoidance (Rangel et al., 2008). These responses can be innate (e.g., avoidance of predator odors) or learned through experience. Owing to their simplicity, Pavlovian behavioral tasks have been invaluable in studying the neural basis of reward learning. One of the most celebrated examples at the intersection of reinforcement learning and neuroscience—that midbrain dopamine neurons encode reward prediction errors—was discovered in monkeys performing Pavlovian behaviors (Schultz, Dayan, & Montague, 1997). The simplicity of Pavlovian tasks is also limiting, particularly if there is a mismatch between the innate Pavlovian behavior and the response needed to obtain reward (e.g., withhold an action to obtain reward; Dayan, Niv, Seymour, & Daw, 2006).

2.2 Habitual systems

Habitual behavior describes the mapping of a large range of arbitrary motor responses to stimuli, through repeated reinforcement. Habit systems in the brain learn through trial-and-error over relatively long timescales (Balleine & O'Doherty, 2010). Once learned, values are thought to be cached and behavior can be carried out "automatically." As such, habits are thought to be computationally cheap but inflexible (Daw, Niv, & Dayan, 2005). It can also be difficult to unlearn habitual behavior. This particular feature is the rationale for outcome devaluation, a gold-standard test of habitual behavior. In outcome devaluation experiments, animals are first trained to associate a stimulus with a response (e.g., pressing a lever in response to a tone) to receive a reward. Following training, the outcome is devalued either by pairing with sickness or making the outcome freely available before the task. Animals are then tested to see if they respond to the stimulus. Continued behavioral responses are consistent with habitual behavior (Balleine & O'Doherty, 2010).

2.3 Goal-directed systems

Goal-directed systems are responsible for flexible decision making. They are thought to compute the outcomes associated with particular actions on fast timescales. As such, they are sensitive to changes in environmental contingencies. Outcome devaluation is also used to test the contribution of goal-directed systems to behavior (Balleine & O'Doherty, 2010).

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[b] Although it is commonly stated that Ivan Pavlov used a bell as a stimulus, this story appears to be apocryphal (Cambiaghi & Sacchetti, 2015).

A reduction in responding is taken as evidence that behavior was driven largely by goal-directed systems. This chapter focuses on delineating the contributions of goal-directed systems to flexible decision making. We will focus largely on behaviors, primarily because, in contrast to studies of sensory decision making, it is paramount to have a controlled behavior, in which the experimenter can quantify hidden variables, such as value.
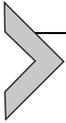
3. Matching behavior

Several behavioral tasks have been used to study flexible decision making: outcome devaluation, reversal learning, set shifting, foraging, "mixed-strategy" games, and matching behavior. Matching behavior (or the matching law) is a type of behavior that we will argue provides excellent conditions to study continual value-based learning. In matching tasks, animals freely choose among two or more options to harvest reward (Herrnstein & Heyman, 1979). Matching describes the tendency of animals to "match" the fraction of choices to a particular option with the fraction of rewards received from that option. Mathematically, matching describes the following relationship:

$$\frac{c_i}{\sum_{i=1}^{N} c_i} = \frac{r_i}{\sum_{i=1}^{N} r_i} \tag{1}$$

where c_i is the number of choices allocated to option i and r_i is the number of rewards obtained from option i, given N possible options. Matching behavior was first observed in pigeons (Herrnstein, 1961) and has since been observed in mice (Bari et al., 2019; Fonseca, Murakami, & Mainen, 2015), rats (Gallistel, Mark, King, & Latham, 2001; Graft, Lea, & Whitworth, 1977), monkeys (Lau & Glimcher, 2005; Sugrue et al., 2004; Tsutsui et al., 2016), and humans (Pierce & Epling, 1983). Matching behavior is typically highly dynamic, with animals switching between options on fast timescales. In trial-based tasks, animals typically switch from one option to another with a mode of 1 trial. However, animals remain reward sensitive, and repeat recently rewarded choices. As such, it can be considered a form of goal-directed behavior.

3.1 Task conditions

Experimental psychologists use tasks in which reward delivery is contingent on schedules of reinforcement (Ferster & Skinner, 1957). Two commonly

used schedules are called "variable ratio" and "variable interval." Matching behavior is classically observed in tasks with variable interval schedules.

Variable ratio schedules are intuitive—reward is simply delivered with a fixed probability, much like flipping a (biased) coin. If given the choice between two variable ratio schedules, the optimal policy is to choose the higher probability option exclusively. Note that exclusively choosing one option is trivially consistent with matching behavior since all choices are allocated to one option, and all rewards are received from that option. Variable-ratio-schedule tasks are typically called "two-armed bandit" tasks and tasks with changing probabilities are called "dynamic two-armed bandit" tasks or, if the probabilities reverse, "probabilistic reversal learning" tasks. In variable interval schedules, reward is delivered after a fixed time has elapsed. Once the time has elapsed, that option is "baited," guaranteeing reward delivery once chosen. This feature is thought to make these tasks ethologically relevant to study foraging. Although this seems like a trivial change, it changes the optimal policy significantly, which will be expanded on below. Intuitively, if given the choice between two variable interval schedules, it does not make sense to choose one option exclusively. Instead, one should occasionally probe the lower-probability option, to not miss out on a baited reward. In these circumstances, matching behavior emerges. In modern, trial-based tasks, discrete versions of variable interval schedules are used, to allow for independent control over inter-trial intervals.

Variable ratio and variable interval schedules are not necessarily different categories but may be thought of as two extremes of a competitive foraging environment (Sakai & Fukai, 2008a). Under this interpretation, "baited" rewards are withdrawn with a particular probability. Variable interval schedules describe environments where the withdrawal probability is 0, imitating herbivore foraging environments without competitors. Intermediate withdrawal probabilities between 0 and 1 imitate competitive foraging environments where food may be intercepted by competitors. Variable ratio schedules are ones where the withdrawal probability is 1, which may resemble the foraging of carnivores.

3.2 Matching behavior is generally a suboptimal probabilistic policy

Given the task conditions that engender matching behavior, a natural question to ask is whether animals are matching because they are aware of baiting (i.e., accounting for environmental statistics) or are they agnostic to it? A key insight into this problem was developed by Sakai and Fukai (2008b), which we expand on below.

For simplicity, we will assume two choices (options a and b). Matching Eq. (1) then reduces to the following form

$$\frac{c_a}{c_a + c_b} = \frac{r_a}{r_a + r_b} \quad \text{and} \quad \frac{c_b}{c_a + c_b} = \frac{r_b}{r_a + r_b} \tag{2}$$

Matching occurs when the relative fraction of choices to option i, c_i, "matches" the relative fraction of rewards, r_i, from option i. Rearranging either equation gives $\frac{r_a}{c_a} = \frac{r_b}{c_b}$. In other words, matching occurs when the mean reward from all options is equated. Written compactly,

$$\hat{r}_a = \hat{r}_b \tag{3}$$

The average reward from both options can be written as

$$\hat{r} = \hat{r}_a \pi_a + \hat{r}_b \pi_b \tag{4}$$

where π_i is the probability of choosing option i. Because we have two options, $\pi_b = 1 - \pi_a$. We assume that the policy is controlled by a parameter x, yielding[c]

$$\hat{r}(x) = \hat{r}_a(x)\pi_a(x) + \hat{r}_b(x)\pi_b(x) \tag{5}$$

The parameter x explicitly influences the policy and implicitly influences the reward probabilities. Under variable-interval schedules, the longer one has stayed away from an option, the higher the probability of reward when that option is eventually selected[d]

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[c] For example, assume $\pi_a(x)$ is a softmax function: $\pi_a(x) = \frac{1}{1 + e^{-x}}$.

[d] The mean reward obtained is a function of x. In variable-interval tasks, this is correct—the animal's policy influences whether it is able to take advantage of the baiting rule. The longer the animal abstains from choosing an option, the greater the probability of reward when the animal chooses it. To express the baiting rule mathematically, we write the probability of reward from option i as

$$P_i(t) = 1 - (1 - p_i)^{t+1} \tag{6}$$

where p_i is the base reward probability and t is the number of consecutive trials since that option was last chosen. This equation states that the probability of reward asymptotically grows from p_i to 1 the longer option i is unchosen. The mean return from option i can be written as

$$\hat{r}_i(x) = \sum_{t=0}^{\infty} (1 - \pi_i(x))^t \pi_i(x) P_i(t) = \sum_{t=0}^{\infty} (1 - \pi_i(x))^t \pi_i(x) \left(1 - (1 - p_i)^{t+1}\right)$$

$$= \frac{p_i}{\pi_i(x) + p_i(1 - \pi_i(x))} \tag{7}$$

These equations can be used to test whether matching is optimal, in closed form.

To maximize reward, we take the derivative of $\hat{r}(x)$ with respect to x and set it equal to 0.

$$
\begin{aligned}
\frac{d\hat{r}(x)}{dx} &= 0 \\
&= \left(\hat{r}_a(x) \frac{d\pi_a(x)}{dx} + \hat{r}_b(x) \frac{d\pi_b(x)}{dx} \right) \\
&\quad + \left(\frac{d\hat{r}_a(x)}{dx} \pi_a(x) + \frac{d\hat{r}_b(x)}{dx} \pi_b(x) \right)
\end{aligned}
\tag{8}
$$

This equation defines the optimal probabilistic policy. The first set of terms in parentheses is the explicit change in behavior when x is changed. The second set of terms is the implicit change in the environment brought about by the animal's policy. One may hypothesize that this first computation (explicit change in behavior) is easy for the brain to perform while the second computation (implicit change in environment) is much more difficult. Sakai and Fukai's critical insight was to recognize that the brain might ignore this second computation, which yields the reduced form

$$
\frac{d\hat{r}(x)}{dx} = 0 = \hat{r}_a(x) \frac{d\pi_a(x)}{dx} + \hat{r}_b(x) \frac{d\pi_b(x)}{dx}
\tag{9}
$$

It can be shown that solving Eq. (9) yields matching behavior. In other words, the solution is when $\hat{r}_a = \hat{r}_b$. Under what conditions is matching optimal?

Fig. 1 provides a geometric intuition for how the policy influences reward in standard matching paradigms. Both options follow variable interval schedules ($p_a = 0.4; p_b = 0.1$). Light and dark blue illustrate $\hat{r}_a$ and $\hat{r}_b$, respectively, as a function of the policy. The consequences of baiting are clear. The less often option a is chosen, the greater the probability of reward when it is chosen. The orange trace illustrates $\hat{r}$ which, from Eq. (4), is the reward rate from both options, weighted by the probability of choosing each option. The optimal reward is at the maximum of this function, which occurs at P(choice to option a) ≈ 0.86. Matching behavior, when $\hat{r}_a = \hat{r}_b$ (Eq. 3), occurs when the two blue traces intersect one another. In standard matching paradigms, matching *is* the optimal probabilistic policy. However, in this circumstance matching can occur either if animals are aware of environmental statistics (Eq. 8) or if they ignore it (Eq. 9).

There is a strong-inference experiment that can test whether animals will continue to exhibit matching behavior, even when it is suboptimal. If they
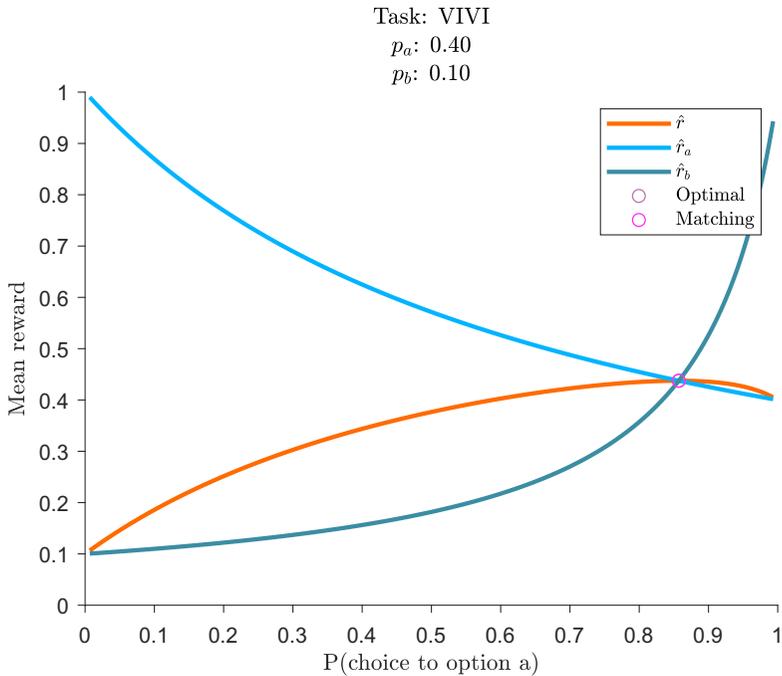
Fig. 1 Matching is the optimal probabilistic policy in variable-interval/variable-interval tasks.

continue to show matching behavior, this is evidence that they likely do not take into account environmental statistics to make decisions. If they show optimal behavior, then they likely are. The key experiment is to only let one option follow a variable-interval schedule. The second option should deliver reward with a fixed reward probability (variable–ratio schedule). This example is illustrated in Fig. 2 ($p_a = 0.3$, variable–ratio option; $p_b = 0.24$, variable-interval option). It is clear that option a, the variable ratio option, does not benefit from baiting. No matter how often it is chosen, the probability of reward is fixed. In this task, matching is not the optimal solution. When these types of tasks have been tested, matching behavior has been observed (Herrnstein & Heyman, 1979; Vyse & Belke, 1992; Williams, 1985), including in humans (Savastano & Fantino, 1994). Under the theory proposed by Sakai and Fukai, these findings indicate that animals behave as if they are not aware of environmental statistics. This should not be taken as evidence that animals are *unable* to calculate these environmental statistics—simply that under these task conditions, they behave as if they
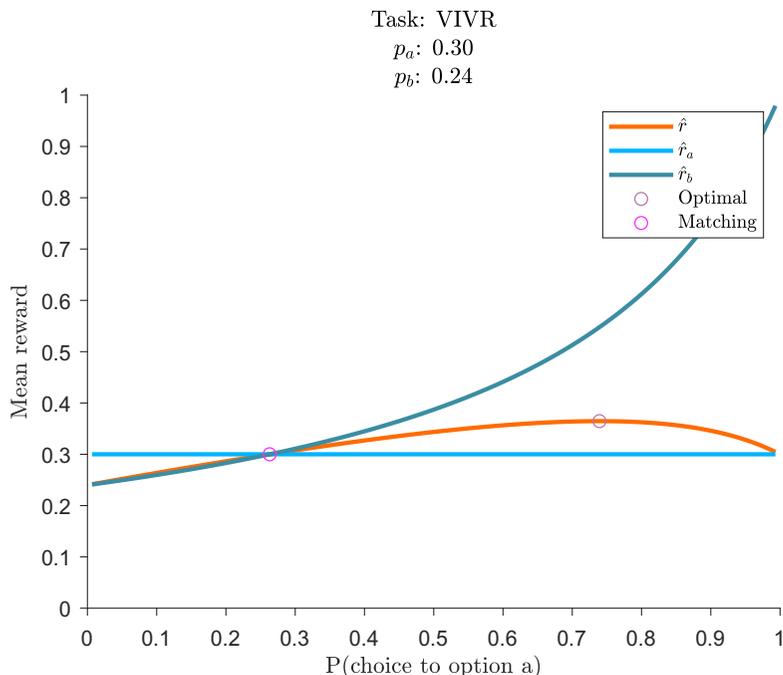
Fig. 2 Matching is not optimal in variable-interval/variable-ratio tasks.

do not. Manipulations designed to encourage optimal behavioral can be successful—for example, rewards that differ in magnitude rather than probability, and allowing practice without reinforcement (Tunney & Shanks, 2002).

3.2.1 Animals do not adopt deterministic switching policies

One limitation of the argument above is that it is limited to probabilistic policies. Deterministic switching policies (i.e., sample the other arm every n choices) are the true optimal policies. For example, if given two variable-interval options in which the base probabilities are $p_a = 0.4$ and $p_b = 0.1$, the optimal policy is to choose option a four times and choose option b once. This is because, according to Eq. (6), after not being selected for four consecutive trials, the probability of reward from option b has climbed from 0.1 to 0.4095, at which point the probability of reward is greater than option a, and it should be chosen. After option b has been chosen, the probability of reward drops again to 0.1, the probability of reward on option a is now 0.64 (since it has not been chosen for one trial), and option a should be chosen.

These policies are easy to diagnose, since stay duration histograms will not display a characteristic exponential shape. Under the hypothesis that animals are adopting a deterministic switching policy, stay durations should instead display a bimodal shape. We are not aware of any human or animal studies demonstrating such behavior, although a computational study trained artificial neural networks which exhibited these switching policies in certain conditions (Wang et al., 2018).

3.2.2 Matching behavior vs probability matching

Matching behavior should *not* be confused with probability matching. Matching behavior and probability matching are observed in very different task conditions. Probability matching refers to the tendency of subjects to match the relative fraction of choices to the probability of reward in two-armed bandit tasks (Mongillo, Shteingart, & Loewenstein, 2014). In two-armed bandit tasks, unlike matching behavior tasks, reward probability does *not* depend on past choices—the options follow variable-ratio schedules. For example, if $p_a = 0.75$ and $p_b = 0.25$, probability matching occurs when subjects choose option *a* 75% of the time and option *b* 25% of the time. This is clearly a suboptimal policy, since the subject should choose option *a* 100% of the time.

Mathematically, probability matching can be written as

$$\frac{c_a}{c_a + c_b} = \frac{p_a}{p_a + p_b} \tag{10}$$

where p_i is the probability of reward associated with option *i*. Unlike matching behavior, there is no circumstance in which probability matching is optimal. Intuitively, in a two-armed bandit task, the optimal policy is to exclusively choose the high-probability option. To see more rigorously how matching behavior and probability matching are incompatible, note that the actual reward received from option *i* is $r_i = p_i \cdot c_i$. Matching behavior is therefore

$$\frac{c_a}{c_a + c_b} = \frac{r_a}{r_a + r_b} = \frac{p_a \cdot c_a}{p_a \cdot c_a + p_b \cdot c_b} \tag{11}$$

With fixed reward probabilities, matching behavior is obtained when $c_a = 0$ or $c_b = 0$, which is incompatible with probability matching. In two-armed bandit tasks, matching behavior is (trivially) to exclusively choose one option - the optimal policy.

Interestingly, probability matching and *undermatching*, the tendency to behave more randomly than perfect matching behavior, may share the same underlying mechanism. If subjects believe the world is more unstable or prone to change than it truly is, both of these phenomena can emerge (Shanks, Tunney, & McCarthy, 2002; Yu & Cohen, 2009; Yu & Huang, 2014).

3.3 Dynamic foraging tasks

In addition to the ethological relevance of matching behavior, a useful feature is the tendency of animals to exhibit highly flexible behavior, switching from one option to another on short timescales. However, one limitation of classic matching paradigms is reward contingencies are typically fixed within sessions and varied across sessions. Practically, both trial–by–trial algorithmic studies of behavior and neurophysiologic studies of flexible behavior benefit from task designs that better capture the full dynamic range of matching within session. Take the example in Fig. 3. Matching behavior is typically illustrated in these types of plots, with the relative reward ratio on the x-axis and choice ratio on the y-axis. Imagine a single session with fixed reward
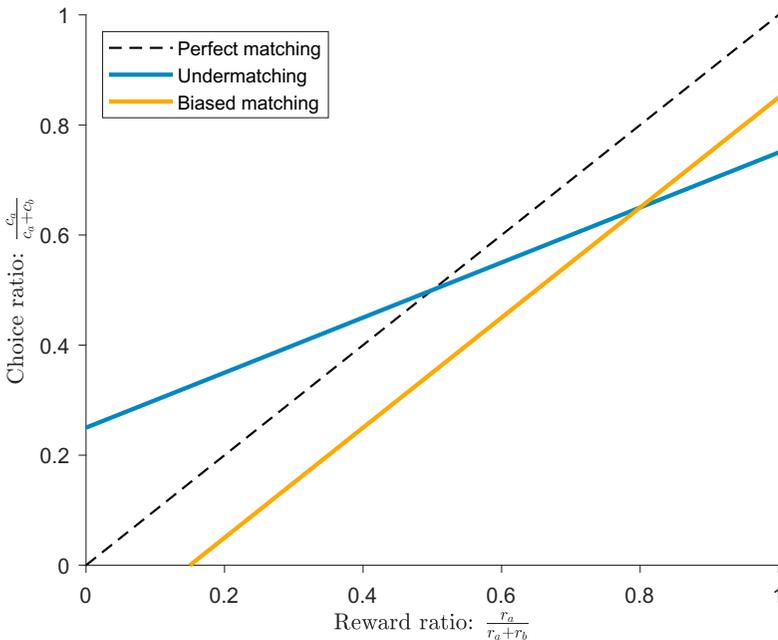

Fig. 3 Dynamic foraging tasks allow for better characterization of behavior.

contingencies that yield a reward ratio of $\sim$0.8 and a choice ratio of $\sim$0.6. This single data point would be consistent with both undermatching (the blue curve; tendency to behave more randomly than perfect matching) and biased matching (the orange curve; tendency to prefer one choice more than another). Since these two hypotheses have very different algorithmic and neural underpinnings, it is useful to use a task design that allows one to measure along multiple choice/reward ratios. Dynamic foraging tasks are variable–interval/variable–interval tasks with multiple reward contingency changes in one session (Gallistel et al., 2001; Lau & Glimcher, 2005; Sugrue et al., 2004; Tsutsui et al., 2016). This task variant elegantly solves this problem.
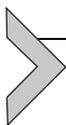
4. Algorithms underlying matching behavior

Matching is a description of macroscopic, averaged behavior. An important question therefore is how does matching behavior emerge from trial–by–trial behavior?

4.1 Melioration

Melioration is among the earliest trial-by-trial algorithms developed to solve this problem (Herrnstein & Vaughan, 1980). This algorithm states that behavior should tend toward the option with the highest local rate of reinforcement (the highest $\frac{r_i}{c_i}$ ratio), which yields matching behavior in the limit. This algorithm was studied extensively by Herrnstein and others, largely to contrast with theories of optimal decision making (Herrnstein, 2000).

4.2 Local matching

Local matching is another algorithm designed to yield matching behavior (Sugrue et al., 2004). Under local matching, the agent exponentially integrates reward from each option and maps the *local* reward ratio to a probability of choice.

$$\frac{\hat{r}_a}{\hat{r}_a + \hat{r}_b} = P(c_a) \tag{12}$$

where $\hat{r}_i$ is the local estimate of reward. For example, $\hat{r}_i$ can be updated as

$$\hat{r}_i = \hat{r}_i + \alpha(R - \hat{r}_i) \tag{13}$$

each time option i is chosen (where R is reward). Although melioration and local matching both do a reasonable job describing behavior, deep insight is limited since both of these algorithms were designed to exhibit matching behavior.

4.3 Covariance-based update rules

A general insight into this problem was developed by Loewenstein and Seung (2006). Assume a change in synaptic weights ΔW is given by the following form

$$\Delta W = \alpha \cdot \mathrm{cov}(R, N) \tag{14}$$

where α is a plasticity rate, R is reward, and N is neural activity. It can be shown that synaptic plasticity update rules of this form converge to matching behavior. Examples of these update rules include

$$\Delta W = \alpha \cdot (R - \mathbf{E}(R))N \tag{15}$$

$$\Delta W = \alpha \cdot R(N - \mathbf{E}(N)) \tag{16}$$

$$\Delta W = \alpha \cdot (R - \mathbf{E}(R))(N - \mathbf{E}(N)) \tag{17}$$

where $\mathbf{E}(X)$ is the expected value. These particular update rules are equivalent to the learning rules in the direct actor and actor-critic reinforcement learning algorithms (Dayan & Abbott, 2001; Sakai & Fukai, 2008a, 2008b). These results demonstrate that matching behavior can be the outcome of very simple learning rules, a remarkably deep insight and one that makes it possible to more confidently interpret neural correlates of value-based decision making in these behaviors.

4.4 Logistic regressions

One of the most common means of analyzing behavior in dynamic foraging tasks is to calculate logistic regressions to predict choice as a function of reward history and choice history (Fonseca et al., 2015; Lau & Glimcher, 2005; Sul, Jo, Lee, & Jung, 2011; Tsutsui et al., 2016). These models take the following form

$$\log\left(\frac{P(c_a(t))}{1 - P(c_a(t))}\right) = \sum_{i=1}^{N} \beta_i^R (R_a(t-i) - R_b(t-i))$$

$$+ \sum_{i=1}^{N} \beta_i^c (c_a(t-i) - c_b(t-i)) + \beta_0 \tag{18}$$

where

$$R_a(t) = \begin{cases} 1, & \text{if option } a \text{ was rewarded} \\ 0, & \text{if either option was not rewarded} \end{cases} \tag{19}$$

$$c_a(t) = \begin{cases} 1, & \text{if option } a \text{ was chosen} \\ 0, & \text{if option } b \text{ was chosen} \end{cases} \tag{20}$$

and vice versa for $R_b(t)$ and $c_b(t)$.

These particular models are powerful since they can capture arbitrary linear combinations of reward and choice history (up to N trials into the past) to predict upcoming choices. A frequent observation is that choices have a positive dependence on reward history (β_i^R coefficients), often multiple trials into the past. The interpretation is that previous rewarded choices reinforce future choices to that option. Previous choices (generally 1–2 trials) tend to have negative coefficients (β_i^c coefficients), meaning animals tend to switch their choices over short timescales, regardless of reward history. Interestingly, negative β_i^c coefficients are generally not seen in tasks without baiting (Parker et al., 2016). Given the arguments above suggesting that animals behave as if they are *not* aware of the baiting rule, one potential explanation is that animals are implementing a simple "switch" heuristic to increase reward rate.

A major limitation of the logistic regression formulation is that only linear combinations of the input can be used. An immediate consequence of this can be seen by observing that when $R_a(t) - R_b(t) = 0$, the model cannot discriminate whether it was option a that was not rewarded or option b. A solution to this problem may exist, since it has been shown that the logistic regression model is identical to an action–value reinforcement learning model with identical learning and forgetting rates (Katahira, 2015). This suggests that the action–value reinforcement learning model may be a suitable template to build interpretable algorithms that capture the essence of matching behavior.

4.5 Action-value reinforcement learning algorithm (Q-learning)

The action–value reinforcement learning (or Q-learning) algorithm is a general-purpose learning algorithm that keeps track of the values (i.e., expected future reward) of available actions and makes decisions based on the difference between action values (Watkins & Dayan, 1992). It has been widely applied both in neuroscience (Akam et al., 2017; Li & Daw, 2011;

Samejima, Ueda, Doya, & Kimura, 2005; Sul, Kim, Huh, Lee, & Jung, 2010) and machine learning (Mnih et al., 2015). If we assume a task with two actions, a common implementation is to assume a single state and update the Q-values as follows. If action on trial $t = a$

$$Q_{t+1}(a) = Q_t(a) + \alpha(r - Q_t(a)) \tag{21}$$
$$Q_{t+1}(b) = Q_t(b) \tag{22}$$

The difference between Q-values is used as an input into a softmax function to produce the probability of a choice.

$$P(\text{action on trial } t = a) = \frac{1}{1 + e^{-\beta(Q_t(a) - Q_t(b))}} \tag{23}$$
$$P(\text{action on trial } t = b) = 1 - P(\text{action on trial } t = a) \tag{24}$$

Forgetting can be introduced as (where action on trial $t = a$)

$$Q_{t+1}(a) = \zeta Q_t(a) + \alpha(r - Q_t(a)) \tag{25}$$
$$Q_{t+1}(b) = \zeta Q_t(b) \tag{26}$$

Forgetting can be applied to just the unchosen action or both actions (Bari et al., 2019; Farashahi, Rowe, Aslami, Gobbini, & Soltani, 2018; Hattori, Danskin, Babic, Mlynaryk, & Komiyama, 2019; Katahira, 2015).

4.6 Bayesian inference

Bayesian inference algorithms use Bayes rule to iteratively update estimates of reward probability. They are powerful since they estimate full probability distributions, allowing them to make decisions that take into account uncertainty in their estimates, as well as higher-order moments. For example, if the algorithm estimates both options to have a mean reward probability $p_a = p_b = 0.5$, but the uncertainty of $p_a > p_b$, then it is adaptive to choose p_a since the true mean reward rate might be higher, increasing reward in the long term (Sutton & Barto, 1998). These algorithms have been used to argue that matching might occur due to uncertainty about changing reward dynamics (Yu & Huang, 2014). In general, they have seen limited use for quantifying matching behavior, since simpler models often do well enough.
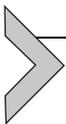
5. Movement vigor during flexible decision making

Decisions are much more complex than just discrete choices. Behavior occurs in real-time and the nervous system must finely calibrate

the vigor of movements. Vigor, often defined as reaction time plus speed of movements, has long been studied in the context of motor control (Choi, Vaswani, & Shadmehr, 2014; Reppert et al., 2018; Rigas, Komogortsev, & Shadmehr, 2016). Recently, vigor has been appreciated as a reflection of value (Shadmehr, Reppert, Summerside, Yoon, & Ahmed, 2019). Vigor is increased by increasing reward (Summerside, Shadmehr, & Ahmed, 2018), decreased by increasing effort (Stelmach & Worringham, 1988), and modulated on short, individual–decision timescales (Reppert, Lempert, Glimcher, & Shadmehr, 2015).

From a normative perspective, modulating vigor in relation to value/ effort is an appropriate computation (Niv, Daw, Joel, & Dayan, 2007; Yoon, Geary, Ahmed, & Shadmehr, 2018). Because increased vigor requires greater energy (Hoyt & Taylor, 1981; Ralston, 1958; Selinger, O'Connor, Wong, & Donelan, 2015), it is not always appropriate to move with high vigor. However, in the context of a highly rewarding environment, it can be worth increasing vigor to increase reward rate, since slow movements necessitate a longer time between movement initiation and receipt of reward. It is clear that the brain modulates movement vigor to maximize reward rates (Haith, Reppert, & Shadmehr, 2012).

In the context of flexible decision making, moment–to–moment vigor has been shown to be as flexible as choice–based behavior. One study employed a dynamic two–armed bandit task in rats and found that latency of task initiation was highly correlated with instantaneous probability of reward (Hamid et al., 2016). Movement vigor was most strongly modulated by the reward rate of the environment (how much reward can be expected regardless of action), and much less so by relative reward rates (how much better one option is relative to another). A number of studies have demon- strated that recent reward history modulates movement vigor (Bari et al., 2019; Del Arco, Park, Wood, Kim, & Moghaddam, 2017; Ottenheimer et al., 2020; Simon, Wood, & Moghaddam, 2015).
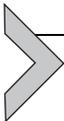
6. Brain structures underlying flexible behavior

Extensive work, most of it in humans, has correlated changes in activ- ity in multiple brain regions with variables from reinforcement-learning models (e.g., Daw, O'Doherty, Dayan, Seymour, & Dolan, 2006; Doya, 2008). The vast majority of these studies have focused on brief changes in activity of neurons or fMRI signal (seconds or less) and model variables that change on similar timescales.

One key variable, common to many algorithms, is the reward prediction error, the difference between actual and predicted reward. We know much about these signals, especially in the context of foraging (Morris et al., 2006; Parker et al., 2016). However, we know much less about the representations of the decision variables updated by these reward prediction errors. In particular, all algorithms of flexible behavior require memory: a summary of previous interactions with the environment that allows for adaptive behavior in the future. Whereas this memory signal—typically in the form of action values or their arithmetic combinations—has been observed in several brain structures, most notably the dorsal striatum (e.g., Samejima et al., 2005), it is largely transient and occurs around the time of cues and actions. It is less clear where these memory signals reside in between bouts of interaction with the environment.

6.1 Cortico-basal-ganglia loops: Medial prefrontal cortex and dorsomedial striatum

The medial prefrontal cortex (mPFC) and its downstream target, the dorsomedial striatum, have long been studied as critical components for generating flexible behavior[e] One view of this circuit is that cortex provides signals to bias action selection (Murakami, Shteingart, Loewenstein, & Mainen, 2017) and the striatum is responsible for action selection itself. Cortical circuitry is predominantly recurrent, which is hypothesized to allow for integration of information which can be routed to striatum to bias action selection. Striatal circuitry, in contrast, is predominantly inhibitory and weak, with lateral inhibition motifs, facilitating a winner–take–all operation (Morita, Jitsev, & Morrison, 2016). The classic view of the striatum considers two parallel cortico-striato-cortical loops, operating to initiate and inhibit actions (Fig. 4). The "Direct" or "Go" pathway, consists of cortical inputs to D1-expressing medium spiny neurons in the striatum, which synapse onto globus pallidus pars interna (entopeduncular nucleus in the rodent (Grillner & Robertson, 2016; Wallace et al., 2017)) and the substantia nigra pars reticulata. These in turn synapse onto the thalamus, and back to the cortex, completing the loop. The "Indirect" or "No-Go" pathway consists of cortical inputs synapsing onto D2-expressing medium spiny neurons,

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[e] In rodents, mPFC typically refers to (from dorsal to ventral) the anterior cingulate cortex, the prelimbic cortex, the infralimbic cortex, and the medial orbital cortex. Dorsal mPFC usually means anterior cingulate cortex and prelimbic cortex, and ventral mPFC usually means infralimbic cortex and medial orbital cortex. In general, since the study of mPFC is still nascent, especially in mice, stereotactic coordinates are useful for comparing studies.
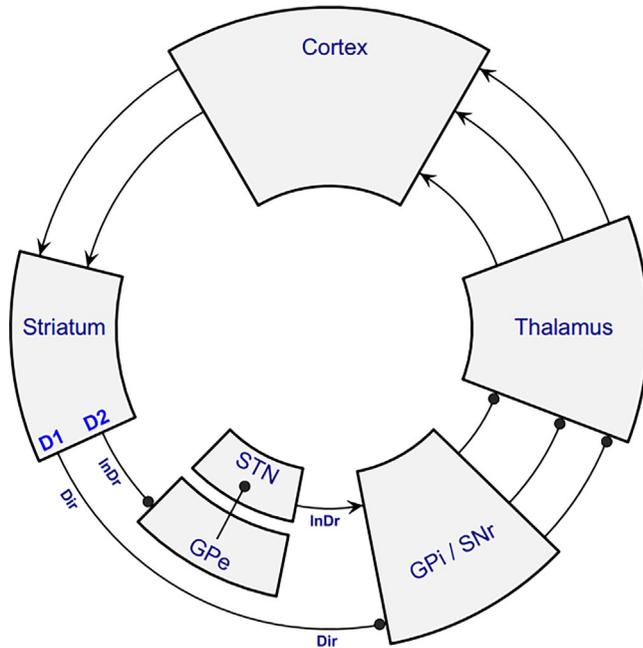
Fig. 4 Classic view of the "Direct" and "Indirect" striatal pathways. *Adapted from Shipp, S. (2017). The functional logic of corticostriatal connections. Brain Structure and Function, 222(2), 669–706. https://doi.org/10.1007/s00429-016-1250-9.*

which in turn synapse onto the globus pallidus pars externa → subthalamis nucleus → globus pallidus pars interna and substantia nigra pars reticulata → thalamus, and finally back to cortex (Shipp, 2017).

Lesions of either the medial prefrontal cortex or the dorsomedial striatum are known to abolish goal-directed behavior and render behavior under the control of sensorimotor associations (Balleine, Delgado, & Hikosaka, 2007; Kennerley, Walton, Behrens, Buckley, & Rushworth, 2006). Neurons in rat dorsal mPFC predict upcoming outcomes, before they have been presented (Del Arco et al., 2017), and monitor action/outcome contingencies (Hyman, Whitman, Emberly, Woodward, & Seamans, 2013; Simon et al., 2015; Sul et al., 2010). Neurons in primate medial frontal cortex signal prediction errors of action values (Matsumoto, Matsumoto, Abe, & Tanaka, 2007) and those in anterior cingulate cortex encode reward history as well as reward prediction errors[f] (Seo & Lee, 2007). In humans, fMRI studies

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[f] Comparisons between rodent and primate frontal structures should be taken with a grain of salt, as anatomical homologies between these orders is weak (Uylings, Groenewegen, & Kolb, 2003).

have revealed a role for medial frontal regions in encoding reward magnitude and value of chosen actions (Daw et al., 2006). Higher network coordination between cortex and striatum predicts changes in learning and decision making (Gerraty et al., 2018).

Recordings in the primate caudate, a homolog of the rodent dorsomedial striatum, have demonstrated encoding of action values and chosen values, key signals in reinforcement learning models of behavior (Lau & Glimcher, 2008; Samejima et al., 2005). Later studies discovered preferential encoding of the difference of temporally discounted values, and encoding of future actions (Cai, Kim, & Lee, 2011). Recordings in the dorsomedial striatum of the rat have confirmed these findings (Ito & Doya, 2009, 2015; Kim, Lee, & Jung, 2013; Seo, Lee, & Averbeck, 2012), and extended them by demonstrating encoding of total value, necessary for modulating vigor (Wang, Miura, & Uchida, 2013). Manipulation of striatal subpopulations modulates choice-based behavior in a manner consistent with changes in action values (Tai, Lee, Benavidez, Bonci, & Wilbrecht, 2012). Unilateral activation of D1-expressing medium spiny neurons in the dorsomedial striatum bias animals to make more contralateral actions. Conversely, unilateral activation of D2-expressing neurons in the dorsomedial striatum bias animals to make more ipsilateral actions. Similar results were obtained with pharmacological manipulation of D1/D2 receptors in the primate putamen (Ueda et al., 2017). These findings align with the classic view that D1-expressing neurons are organized into the "Go" pathway to initiate behavior, and D2-expressing neurons are organized into a "No-Go" pathway to inhibit behavior.

Recent work has focused on mouse mPFC, a structure known to have persistent working-memory-like neural correlates (Liu et al., 2014), as the potential site where decision variables are maintained in between bouts of interaction with the environment. Persistent activity is a viable network mechanism for maintaining representations of decision variables in the times between choices. Persistent activity, defined here as activity that lasts longer than the time constants of synaptic inputs, was first described in the prefrontal cortex and mediodorsal thalamus of primates and is thought to be a critical component of working memory (Fuster & Alexander, 1971).

We recently recorded from mPFC neurons, including those that projected to dorsomedial striatum, in mice performing a matching task (Bari et al., 2019). Individual mPFC neurons showed persistent representations of two key decision variables for matching behavior: relative value—used to bias choices—and total value—used to bias response time (or "vigor").

Remarkably, these two forms of persistent activity showed different rates of decay, that matched the behavior they supported. Relative-value activity did not appear to decay during long inter-choice intervals (tens of seconds); neither did the mouse's memory of its choice policy (Fig. 5). In contrast, total-value activity decayed slowly over long inter-choice intervals; likewise, mice made slower choices after long waiting times. These variables did not appear to be robustly encoded by tongue premotor neurons. These data suggest that cortico-basal-ganglia-thalamo-cortical loops maintain value-based decision variables, and that information flow in the circuit is not a simple linear flow of computations.

Tool development has allowed for precise pathway-specific modulation. In our recent study, we found that inactivation of the mPFC → dorsomedial striatal pathway disrupted choice behavior and slowed vigor, consistent with a disruption of cognitive variables necessary for value-based decision making (Bari et al., 2019). This is similar to a recent study which demonstrated that the mPFC → striatum and mPFC → thalamus pathways were necessary for choice behavior, but not the mPFC → mPFC pathway (Nakayama, Ibañez-Tallon, & Heintz, 2018). Another group demonstrated that the mediodorsal → mPFC pathway, but not the mPFC → mediodorsal thalamus, is necessary for updating understanding about the causal structure of actions, although both were necessary for goal-directed behavior (Alcaraz et al., 2018). This is consistent with the notion that the cortico-basal ganglia system is critical for flexible behavior, since the former, but not the latter, pathway is part of this system.

Taken together, these results highlight the key concept that the brain has dedicated circuitry for different behavioral strategies, and that the rodent dorsal medial prefrontal cortex and downstream dorsomedial striatum are important structures for driving flexible behavior.

6.2 Neuromodulatory systems

Neuromodulatory systems are remarkably unique. These systems are each composed of very small numbers of cells, yet have outputs that arborize to span large volumes of tissue. This feature makes them poised to exert global control over neural states and computations.

Midbrain dopamine is a particularly well-studied neuromodulator. Dopamine neurons in the midbrain encode reward prediction errors, a critical variable in reinforcement learning models of flexible behavior (Bayer & Glimcher, 2005; Cohen, Haesler, Vong, Lowell, & Uchida, 2012;
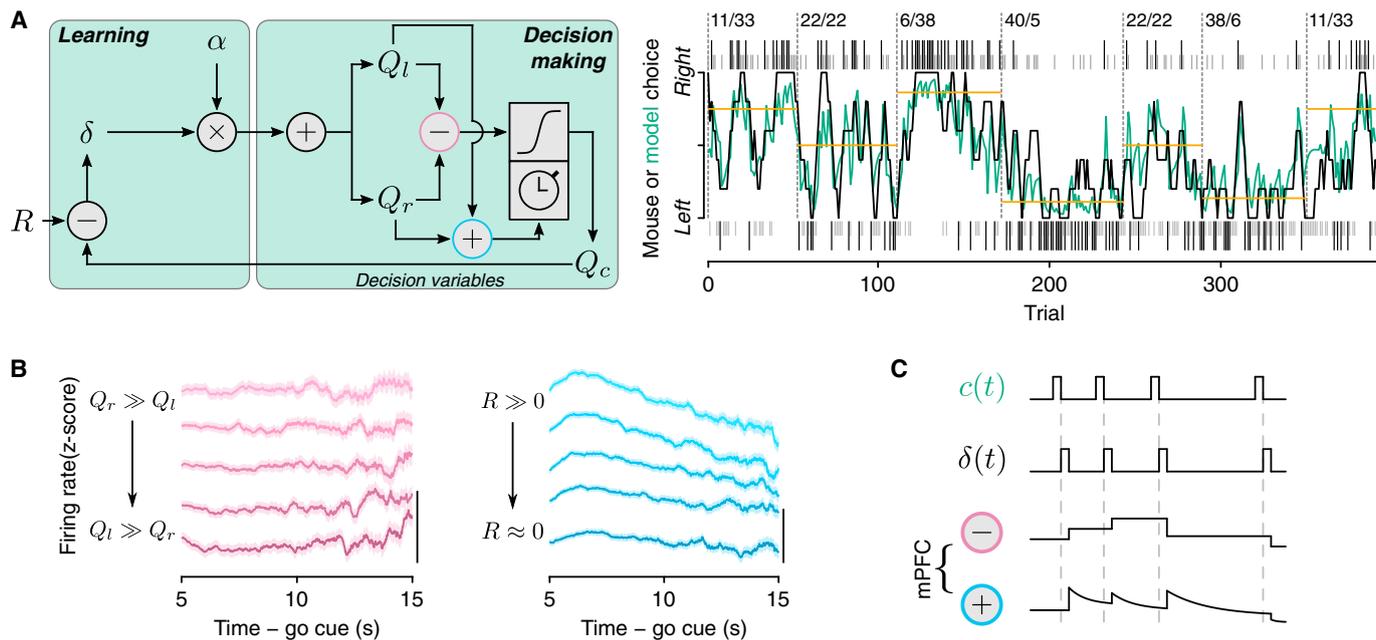
Fig. 5 (A) Reinforcement-learning model (*left*) and fit to example mouse behavior (*right*). (B) Persistent activity representing relative (*left panel*) and total (*right panel*) value. (C) Schematic of variables from the model represented in mPFC. *Adapted from Bari, B. A., Grossman, C. D., Lubin, E. E., Rajagopalan, A. E., Cressy, J. I., & Cohen, J. Y. (2019). Stable representations of decision variables for flexible behavior. Neuron, 103(5), 922–933.*

Morris et al., 2006; Roesch, Stalnaker, & Schoenbaum, 2007; Schultz et al., 1997). Manipulation of the dopamine system profoundly affects learning, for both Pavlovian associations (Steinberg et al., 2013) and for flexible decisions (Hamid et al., 2016; Parker et al., 2016). Dopamine densely innervates the striatum, where it bidirectionally modulates the plasticity of corticostriatal synapses, depending on the downstream receptor (Reynolds & Wickens, 2002). This feature makes dopamine an integral component that enables the cortico-basal ganglia system to modulate flexible behavior.

Serotonin is comparatively less-well understood, in large part to substantial heterogeneity in this system relative to midbrain dopamine. In large thanks to tool development (Lima, Hromádka, Znamenskiy, & Zador, 2009), extracellular recording of single cell-type-identified serotonin neurons has allowed for inroads to be made. Recent work has shown that a subset of serotonin neurons in the dorsal raphe nucleus encodes value over very long timescales (Cohen, Amoroso, & Uchida, 2015), which may relate to the encoding of background reward rate, a key variable in models of optimal foraging behavior (Charnov et al., 1976). Activation of dorsal raphe serotonin neurons in mice performing a dynamic foraging task leads to increases in learning rates (Iigaya, Fonseca, Murakami, Mainen, & Dayan, 2018) and promotes persistence in mice performing a patch foraging task (Lottem et al., 2018). These findings must be interpreted with caution, however, since serotonin manipulation can have opposite effects on behavior, depending on the outputs of distinct subpopulations of neurons (Ren et al., 2018).

Norepinephrine produced by neurons in the locus coeruleus is thought to be critical for behavioral flexibility. These neurons respond to salient events, which has led to the hypothesis that norepinephrine is critical for arousal and attention (Aston-Jones, Rajkowski, & Cohen, 2000; Carter et al., 2010; Harley, 1987; Kety, 1970). Classic work demonstrated that activation of locus coeruleus norepinephrine can alleviate forgetting in a complex maze task (Devauges & Sara, 1991) and facilitate attentional shifts (Devauges & Sara, 1990). Formal theories of norepinephrine suggest that the system encodes unexpected uncertainty (Dayan & Yu, 2006; Yu & Dayan, 2003, 2005) and facilitate exploitation/exploration of task contingencies (Aston-Jones & Cohen, 2005).

6.3 Other structures

Although we have focused heavily on cortico-basal ganglia systems and neuromodulatory systems, flexible behavior relies on a much larger network of

structures. The orbitofrontal cortex, and its upstream/downstream structures, are critical for flexible behavior. Examples include the orbitofrontal cortex → submedius nucleus pathway (Fresno, Parkes, Faugère, Coutureau, & Wolff, 2019) and the amygdala → orbitofrontal cortex pathway (Fiuzat, Rhodes, & Murray, 2017). The orbitofrontal cortex encodes reward magnitude (Simon et al., 2015) and codes for chosen value and reward prediction errors more strongly than mPFC (Sul et al., 2010). Other important structures include the posterior parietal cortex and posteromedial cortex (Funamizu, Kuhn, & Doya, 2016) and the ventral hippocampus (Yoshida, Drew, Mimura, & Tanaka, 2019). This list is by no means exhaustive and serves simply to indicate that large portions of the brain are critical for flexible decision making.
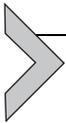
7. Future directions

Despite sophisticated theoretical and modeling insight about flexible decision making, there remains a gap in our knowledge of how the entire loop—sensory signal to discrete action—is instantiated in neural circuitry. One direction that we believe is due for further study is expanding action space to more than two discrete actions, especially using continuous action space. This is likely to enhance our theories of value representation in the mPFC, extending beyond the relatively impoverished regime of binary choices.

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